Lesson Study #4 – Proof of Pythagorean Theorem

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# Part 1: Background Information and Research Goals

Background Information About School and Students

Our group of interns has been placed at Waverly Middle School for the yearlong internship program for Michigan State University. During our experience at Waverly Middle School, our team of three interns has now participated in four lesson studies throughout the school year where each intern has led at least one lesson study. The mentor teacher of the lead intern also participated in each lesson study. These were done to enhance our own instruction practice and give students tasks of increased challenge and engagement to analyze student understanding.

Waverly Middle School is on the west side of Lansing, Michigan. WMS serves students in grades seven and eight. The school has a focus on promoting and respecting the diverse cultures that the student body provides. There are 70% of students who have free lunch. Waverly has numerous programs looking to improve the behavior and success of students. The Response to Intervention (RTI) program providesstudents an Academic Achievement class in area to provide enrichment or assistance. This is done from assessing students on numerous skills. There is special education support as well. The classroom focused on this specific study is team taught with a special education teacher in the classroom. WMS currently has a technology grant incorporating technology in the classroom in multiple forms (SMART Board, Mobi View, iPad, Laptop, etc.). With the amount of technology provided, it is very important to use a variety of instruction incorporating all of these tools.

General Goals for Using Lesson Studies

The use of lesson studies is greatly helpful in the progression of our instructional experience. There are specific goals to be accomplished when using lesson studies in class. Provided below is a list of goals for us as a group:

* Enhance lesson plan skills through collaboration.
* Reflect upon experience of incorporating tasks of high cognitive demand.
* Create a variety of lessons that incorporate tasks that reach all types of students.
* Build a repertoire of lesson tools to use in the future.

When holding us to these goals, it creates expectations for the students that we want to uphold during the lesson study. Students should be held accountable to be able to do the following:

* Actively engage in tasks of higher cognitive demand.
* Discuss the material they are learning with their peers.
* Explain through writing and discussion the mathematics behind their work.
* Explore learning through different styles of instruction.

# **Part 2: Mathematics Overview**

Pythagorean Theorem Unit and Prior/Future Lessons

This lesson on the proof of Pythagorean Theorem comes halfway through the unit focused on Pythagorean Theorem. Prior to this lesson, students had been constructing different squares from each side length of the right triangle. The students had noticed a relationship between the areas of the squares. The sum of areas of the squares from the leg lengths equaled the area of the square constructed from the hypotenuse. Once this relationship was established, the students had made a connection to the side lengths of the right triangle. Then the relationship was provided in a formula and given the name of Pythagorean Theorem. The students then started to practice finding missing side lengths by using the formula and solving for the missing length.

After the proof of Pythagorean Theorem lesson, some time will be spent on the converse of the theorem. This entails determining if just knowing the three side lengths of a triangle could tell the students if the triangle was right. More proof practice would follow with students finding a proof for the converse of Pythagorean Theorem. Once this is accomplished, there will be a few days of exploration of solving real-world situations in which a representation of a right triangle will be necessary to solve the problem. Finally, there will be time to find a missing length on a coordinate grid by constructing a right triangle and finding the length of the hypotenuse.

Lesson Study Goal and Plan for Lesson

This lesson is the first time students will be experiencing proofs in their math instruction. Therefore, it is very important for students to understand the importance of proofs in mathematics. One goal from this lesson study is: **Students can explain why they need to use proofs when they work with math and what constitutes a proof in math.** This is a lofty goal for an introductory portion of the lesson. Therefore, the launch must be structured in a way where students understand the relevance of proofs. Initially, students and I will state the “I Can” statement of the day, which is proving the Pythagorean Theorem. Next, I need to pose what a proof is to students. Understanding that most students may be unsure of what exactly to say, I am prepared to ask my students when they have to prove something in their life. When teaching proofs in middle school, the easy connection can be made to how students have to prove their innocence when talking to parents. Asking questions like, “What do you need to prove yourself?” allows for students to understand what they need in a proof. Once this is established, I want students to think about what makes something a proof. While students consider this, I want them to think about how to prove Pythagorean Theorem. These topics are in the form of an entry slip and will be turned in after the short discussion.

Once a discussion of proofs has launched the task for the lesson, I want to get started with the ultimate goal for the lesson: **The students will prove Pythagorean Theorem through the use of manipulatives.** The students first will be given the puzzle pieces in pairs of two. The students are already sitting in table pairs so there should be no issue with groupings. To start the task, the students will be asked to explore the puzzle pieces. The students must determine what kind of triangles they are working with. Once the students understand that right triangles are being used in the puzzle, they will be asked to find a relationship between the triangles and squares. The use of manipulatives will potentially lead to students finding that each length of the triangle will equal in length when compared to one of the three squares. Students will be writing their findings down on a worksheet to assist them in the future.

The next portion of the lesson will involve students using the puzzle pieces. Given that there are two boards, the students will try to complete the puzzles. Using their knowledge of size of squares, students can use this knowledge to assist them. I anticipate students will struggle to complete the puzzle with the largest square. This is because the square is slanted. I will try to ask questions to guide them in the right direction. This section is completely exploratory and will be done in groups of two. The students will then answer questions over what they used to solve the puzzle. Once the students have reached this point, there is enough material to create a discussion. With the completed puzzles projected for students to see, there will be a discussion leading to the proof. First, the students will need to find the similarities and differences of the puzzle. Using these facts, the students will attempt to draw a conclusion. From all of the discoveries the goal is that students will find Pythagorean Theorem from finding the differences in the area of squares. Finally, there is a discussion on how this actually constitutes a proof. The students will have to reflect on what they stated as a proof at the start of the lesson and show how they have grown over what makes something a proof. Students will be expected to turn in their guided sheet on the Pythagorean Puzzle as a way to assess their work in class. Students will also complete an exit slip to indicate their growth in understanding of proofs from the start of the lesson.

# Part 3: Description of the Lesson

After students were getting accustomed in class, I passed out an entry slip in class. While these were being passed out, I explained to students they would be working with proofs today. I announced the goal for the lesson was for students to say, “I can prove Pythagorean Theorem”. Once this was announced, I launched a discussion on proofs. I asked students what they thought a proof was. Most students were quiet and unresponsive. I was prepared to alter the conversation slightly to get more students engaged. I rephrased the question to ask students if they ever had to prove themselves in their life. Most students still wanted to be too cool for the discussion; so then I altered the question ever so slightly. I told the students I didn’t totally believe them since they were middle school students. I stated that they have to prove themselves in their family all the time. Even making the statement such as, “You’re telling me that your parents never thought you got in trouble?” totally changed the behavior in class. Of course, most students would love to continue to complain how they always get in trouble, so I brought the focus back in the discussion. I asked to the students, “What do you need to do when proving yourself to your parents?” Many students looked towards evidence. Students said they needed evidence to make their parents believe them. I used this as a connection to the next piece of the launch. I asked why proofs are necessary in math. To go along with this, I asked students to wonder what Pythagoras had to do to make other peers believe his theorem. When students started answering with answers surrounding the use of evidence, I progressed to the final part of the launch. The students had to think about how to find evidence in their math. I gave students some time to think about this. As students were thinking about this, they were expected to complete the entry slip. There were some students who hadn’t completed the entry slips, but I proceeded to the exploratory portion of the task.

Students were given the puzzle pieces for the next portion of the lesson. As students received the pieces I also had a worksheet passed out to the students. I indicated there would be multiple parts to the task. I then announced that students need to pull out one triangle and the three square pieces. I asked the students to find a relationship between the triangle and the squares. I left this portion of the task open-ended, anticipating students to play with the pieces and let me know what they notice. As I monitored the progress around the room, there were some students that had expressed what they found using the manipulatives. There were others that were still confused at what to do. I brought the class together and presented student work on the new Apple TV in the classroom from my iPad. I asked the student to explain their work. After having other groups construct the relationship, the students were instructed to finish the questions from the first part of the worksheet. After a few minutes had passed, the students were instructed to start constructing the puzzles. Before they were left to work in pairs, I brought attention to the front of the room and showed studentsthat four triangles should be placed on each puzzle board. I used the doc cam to show this by placing four triangles on each green puzzle board.

After showing this I explained to the students that the placement of the squares was on them. As the students started working on the puzzle, I monitored the work by walking around the room. I often found students completing the first puzzle with the smaller squares. A common misconception quickly surfaced when students were constructing their puzzles. Many pairs were not noticing a difference in sizes between the medium and large. Therefore, their puzzles were coming together but the incorrect squares were used because the size was not distinguishable. I attempted to individually address this with groups, but there were numerous issues with this misconception. I took pictures of student work as groups were finishing their puzzles. There were pictures of correct student work as well as misconceptions. After most groups were finishing up, I called attention to the front to hold a discussion. I referenced that students should be on part three of the worksheet to participate in the discussion. I discussed the misconception focusing on the size of the squares first. I told students there was a difference and showed the overlap on the Doc Cam. After presenting the two correct puzzles, I led the students in a discussion about the similarities and differences between the boards. Once the students noticed the crucial difference between the puzzles in the squares, I tried to move the conversation to the areas of the squares. There was a lack of labeling the squares at the beginning of the lesson. This issueled to a lot of confusion during the discussion. There wasn’t a clear distinction of labeling the squares. The lesson didn’t lead to finding a concrete formula from the side lengths. The students had understood the differences from the squares in the puzzles, but were struggling to understand why to make the leap to the side lengths to find the formula.

We had reached a point in the lesson where I wanted to have students develop an understanding from the discussion questions. I noticed some struggles so I continued to build on the discussion. I believed that the students still didn’t reach the point of understanding yet. I had reached the end of the lesson before we could complete this discussion on the gained knowledge from the task. This discussion was completed, as well as a discussion on why the puzzle task was a way to prove Pythagorean Theorem, on the next day. This discussion would be completed at the beginning of the lesson on the next day. An exit slip that was created for the end of the lesson was also pushed back to the next day as well. The exit slip would still be used as a way to assess student learning even though it was not part of that lesson.

# Part 4: Observation Guide

Based on the goals from our lesson study experience, there were specific observations sought for during the lesson. There were two areas focused upon during observation of the lesson. Was the task accomplishing the lesson study goal? Did the use of multiple instruction techniques enhance student understanding? As observers, we hoped to answer the following questions during the lesson:

Accomplishing the Lesson Study Goal:

* What were we looking for in the lesson?
* Did the students accomplish the goal?
* Did they gain the desired understanding?

Use of Variety of Instruction Techniques:

* During the discussion on “proof,” what real world examples of proof did students come up with?
* Did the manipultaives used in the lesson enhance students’ understanding of the Pythagorean Theorem?
* Were students engaged in the explore task?
* Did students use the picture representation to help them understand the Pythagorean Theorem?

# **Part 5: Post-Lesson Analysis**

After the lesson, our group debriefed over the lesson study. The key discussion points were established for the discussion. We wanted to focus on how to more effectively connect the algebraic proof and the proof using manipulatives to expand students’ understanding of the Pythagorean Theorem. Initially, our discussion focused on the positives that came from the lesson. There was excitement over the discussion that came from the launch. The students were given an opportunity to think about their own life and consider what they have had to prove in their life. The use of manipulatives was great with giving students a challenge in the form of a puzzle. Most students believed they could complete the puzzle and wanted to complete it. The visuals used in the hands-on activity that could be used in this lesson reaches those students who learn best from having viewing the work in front of them. There were numerous positives, which come from this lesson, but there were some adjustments that could have made the lesson great.

When considering the lesson, we established areas to enhance the discussions in class and greatly develop the aspired understanding in class. There were also revisions that could have greatly improved the end result of the lesson. These revisions included:

* Shorten the journal activity down to one question about finding the missing length of a right triangle.
* Give students a chance to explain situations in which they have had to prove something in real life. Allow for a brief discussion to give students a chance to understand the significance and need for “proof”.
* Label the side lengths of the different-colored squares to be a, b, and c respectively.
* State the area of each square from using the labels of the side lengths. Students can visibly see the connection to the Pythagorean Theorem.
* Explicitly state that the blue and purple squares are different sizes. (This was unclear to most students)
* Have a teacher set of manipulatives to use at the doc-cam, giving students something to look at during discussions.
* Have students complete an exit slip after #11 and #12 on the worksheet. This gives students something to assess their understanding upon completion of their work on the task.

After completing the revisions for the lesson, the initial lesson plan was adjusted to account for these changes. This can be found in the document attached on the lesson plan page.